Units

1. Fill in the empty cells in the following table by making conversions between °F, °C and K (kelvin). Show your work. In other words, *do not* use an on-line conversion tool or built-in calculator function except to check that your hand calculations are correct.

T (°C)	10		
T (°F)		10	
T (K)			10

Solution: The conversion formulas are

$$T\,(^\circ\mathrm{F}) = \frac{9}{5}\,T\,(^\circ\mathrm{C}) + 32 \qquad T\,(\mathrm{K}) = T\,(^\circ\mathrm{C}) + 273.15$$

and the reverse conversion for °C to °F is

$$T\,(^\circ\mathrm{C}) = (T\,(^\circ\mathrm{F}) - 32)\,\frac{5}{9}$$

Proceeding down the first column, and using subscripts to indicate the column we compute

$$T_1\,(^\circ\mathrm{F}) = \frac{9}{5}\,(10\,^\circ\mathrm{C}) + 32 = 18 + 32 = 50\,^\circ\mathrm{F}$$

$$T_1\,(\mathrm{K}) = 10\,^\circ\mathrm{C} + 273.15 = 283.15\,\mathrm{K}$$

We will truncate the temperature in kelvin to 293 K to keep an appropriate number of significant digits.

For the second column

$$T_2\,(^\circ\mathrm{C}) = (10\,(^\circ\mathrm{F}) - 32)\,\frac{5}{9} = -22\,\frac{5}{9} = -12.22\,^\circ\mathrm{C}$$

$$T_2\,(\mathrm{K}) = -12.22\,^\circ\mathrm{C} + 273.15 = 260.93\,\mathrm{K}$$

For the third column

$$T_3\,(^\circ\mathrm{C}) = T_2\,(\mathrm{K}) - 273.15 = 10 - 273.15 = -263.15$$

$$T_3\,(^\circ\mathrm{F}) = T_2\,(^\circ\mathrm{C})\,\frac{9}{5} + 32 = -263.15\,(^\circ\mathrm{C})\,\frac{9}{5} + 32 = -473.67 + 32 = -441.7\,^\circ\mathrm{F}$$

Following is the completed table, where all entries have been rounded to three significant figures.

T (°C)	10	−12.2	−263
T (°F)	50	10	−442
T (K)	283	261	10

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2. Ten joule of energy are transfered to a system in 10 minutes. What is the average rate of power transmission?

   Solution: Power is the rate at which energy is transferred

$$\text{power} = \frac{\text{energy transfered}}{\text{time interval}} = \frac{10\,\text{J}}{(10\,\text{min})\left(\dfrac{60\,\text{s}}{\text{min}}\right)} = \frac{1}{60}\,\frac{\text{J}}{\text{s}} = 0.0167\,\text{W}$$

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3. What must be the units of m, c, T and t be in order that

$$mc\frac{dT}{dt} = VI \qquad (1)$$

   has consistent units when V is in volt and I is in amp?

   Solution: The product VI is power in watt, if V is in volt and I is in amp. Thus, the product of $mc\,dT/dt$ must have units of watt. We will use the following units and carry out the product of units in $mc\,dT/dt$, where $[x]$ is read, "units of x".

$$[m][c]\left[\frac{dT}{dt}\right] = \text{kg}\ \frac{\text{J}}{\text{kg}\cdot\text{K}}\ \frac{{}^\circ\text{C}}{\text{s}} = \frac{\text{J}}{\text{s}}\frac{\text{K}}{{}^\circ\text{C}} = \text{W}$$

   Simplification of the ratio K/°C as dimensionless is allowed since the definition of c is energy per kg per *temperature difference* and a difference in temperature on the kelvin scale is the same as a difference in temperature on °C scale because the additive 273.15 term cancels. For example,

$$T_2(\text{K}) - T_1(\text{K}) = [T_2({}^\circ\text{C}) + 273.15] - [T_1({}^\circ\text{C}) + 273.15] = T_2({}^\circ\text{C}) - T_1({}^\circ\text{C}).$$

   Therefore, for consistent units in Equation (1) are

$$m \text{ in kg}, \qquad c \text{ in } \frac{\text{J}}{\text{kg K}} \text{ or } \frac{\text{J}}{\text{kg}\,{}^\circ\text{C}}, \qquad T \text{ in } {}^\circ\text{C or K}, \qquad t \text{ in s}.$$

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Analysis

1. Use an estimate for the mass of water in your fish tank to compute a value of K for your tank.

   Hint: What are the electrical characteristics of the power resistor used in the heater?

   Solution: The hint is to recognize that the resistance of the heater is known, $10\,\Omega$. We can compute the power dissipated by the resistor at V^2/R, where V is the applied voltage, which is also known. Before using those numerical values we need to do some analysis.

   The value of K is measured from data collected from temperature as a function of time. The slope of the $T(t)$ curve is K:

$$K = \frac{dT}{dt} \qquad (2)$$

   The derivative dT/dt also appears in the energy equation

$$mc\frac{dT}{dt} = VI \qquad (3)$$

where m is the mass of water in the system, c is the specific heat of water, T is the temperature of the water, t is time, V is the voltage applied to the heater and I is the current flowing through the heater. Combining Equation (2) and Equation (3) we can solve for K

$$K = \frac{VI}{mc} \tag{4}$$

Now we can replace VI with

$$VI = \frac{V^2}{R}$$

which is helpful because V and R are known. Making the substitution of V^2/R for VI in Equation (4) gives

$$K = \frac{V^2/R}{mc} \tag{5}$$

c is a material property that we can look up. The value of m can be computed from

$$m = \rho\mathcal{V}$$

where ρ is the density of water in the tank and $\mathcal{V}$ is the volume of water in the tank. Making this last substitution we get an expression for K

$$K = \frac{V^2/R}{\rho\mathcal{V}c} \tag{6}$$

in which everything on the right hand side can be computed or, for ρ and c, values can be found in reference materials, including the course notes.

The volume of water in the tank is

$$\mathcal{V} = \frac{\pi}{4}d^2h$$

where d is the diameter of the tank and h is the depth of water in the tank. Using $d = 4\,\text{cm}$ and $h = 6\,\text{cm}$ gives

$$\mathcal{V} = \frac{\pi}{4}\left(\frac{4}{100}\,\text{m}\right)^2\left(\frac{6}{100}\,\text{m}\right) = 7.540 \times 10^{-5}\,\text{m}^3$$

From reference tables, the class notes, or Wikipedia, we find that the relevant properties of water are

$$\rho = 998\,\frac{\text{kg}}{\text{m}^3} \qquad c = 4180\,\frac{\text{J}}{\text{kg}\,^\circ\text{C}}$$

Substituting the known values into Equation (6) gives

$$K = \frac{(12\,\text{V})^2/10\,\Omega}{\left(998\,\frac{\text{kg}}{\text{m}^3}\right)(7.540\times10^{-5}\,\text{m}^3)\left(4180\,\frac{\text{J}}{\text{kg}\,\text{K}}\right)} = 0.0458\,\frac{\text{W}}{\text{J}/^\circ\text{C}} = 0.0458\,\frac{\text{J/s}}{\text{J}/^\circ\text{C}} = 0.0458\,\frac{^\circ\text{C}}{\text{s}}$$

Convert °C/s to °C/min

$$K = 0.0458\,\frac{^\circ\text{C}}{\text{s}} \times \frac{60\,\text{s}}{1\,\text{min}} = 2.7\,\frac{^\circ\text{C}}{\text{min}}$$

The computed value of K is higher by a factor of 3 than values typically measured in the fish tank. In the physical fish tank, the energy gain (and hence temperature rise) of the water is reduced by heat loss to the surroundings and energy gain by the tank and tubing materials.

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2. Given the value of K from the preceding step, how long should the heater be turned on to increase the temperature of the water by $0.5\,°\mathrm{C}$?

Solution: Replacing the time derivative by discrete differences gives

$$K = \frac{dT}{dt} = \frac{\Delta T}{\Delta t}$$

which allows us to solve for ΔT

$$\Delta t = \frac{\Delta T}{K}$$

Substituting the known values gives

$$\Delta t = \frac{0.5\,°\mathrm{C}}{0.0458\,\dfrac{°\mathrm{C}}{\mathrm{s}}} = 10.9\,\mathrm{s}$$

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3. If the volume of water in the fish tank is increased, e.g., by raising the location of the drain hole, the value of K will (pick one)

   a. Increase
   b. Decrease
   c. Not change
   d. not be known from the information given

Solution: Decrease

4. Use analysis to justify your answer to the preceding question.

Solution: Start with the energy equation

$$mc\frac{dT}{dt} = VI$$

and substitute

$$K = \frac{dT}{dt}$$

and solve for K to get

$$K = \frac{VI}{mc}$$

The mass of water in the tank is $m = \rho\mathcal{V}$, where ρ is the water density and $\mathcal{V}$ is the volume of water. Substituting for m gives

$$K = \frac{VI}{\rho\mathcal{V}c}$$

From this equation we see that increasing $\mathcal{V}$ will decrease K.

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5. What is the percent change in K if the volume of the fish tank is increased by 10%? Yes, *this is computable without assuming any numerical values* other than a 0.1 relative change in K.

Hint 1: Express the volume change symbolically as

$$\frac{\mathcal{V}_2 - \mathcal{V}_1}{\mathcal{V}_1} = \alpha \tag{7}$$

where $\mathcal{V}_1$ is the volume before the change and $\mathcal{V}_2$ is the volume after the change, and α is the fractional change in volume. For the given problem, $\alpha = 0.1$, but there is no need to use that numerical value until the end of the analysis.

Hint 2: To answer the question, you will need to find the fractional (or percent) change in K

$$\frac{K_2 - K_1}{K_1} = \beta. \tag{8}$$

Hint 3: Use the energy equation

$$mc\frac{dT}{dt} = VI. \tag{9}$$

where m is the mass of water in the system, c is the specific heat of water, T is the temperature of the water, t is time, V is the voltage applied to the heater and I is the current flowing through the heater.

Answer: 9.1%. You will have to show whether this is an increase or decrease. Also note that $\beta \neq \alpha$, so don't just guess that an increase of volume by 10% leads to a change in K of 10%. That's guessing. Use analysis.

Solution: We need to relate the volume change to a change in K. The K for the tank is

$$K = \frac{dT}{dt} \tag{10}$$

where the slope, dT/dt, is measured while the heater is turned on.

We want to know how K will change with a change in volume of water. The connection is through the energy equation, Equation (9).

We will neglect the effect of water in the pump and tubing, and consider the volume of interest to be only the volume of the tank, $\mathcal{V}$. To find the relationship between K and $\mathcal{V}$, substitute Equation (10) into Equation (9) and solve for K:

$$mcK = VI \quad \Longrightarrow \quad K = \frac{VI}{mc}$$

But, since $m = \rho\mathcal{V}$ we have

$$K = \frac{VI}{\rho\mathcal{V}c} \tag{11}$$

Substitute this expression into Equaton (7),

$$\frac{K_2 - K_1}{K_1} = \frac{\left(\dfrac{VI}{\rho\mathcal{V}_2 c}\right) - \left(\dfrac{VI}{\rho\mathcal{V}_1 c}\right)}{\left(\dfrac{VI}{\rho\mathcal{V}_1}\right)}.$$

but the values of V, I, ρ and c cancel since those terms will not change with a change in tank volume. Therefore

$$\frac{K_2 - K_1}{K_1} = \frac{\dfrac{1}{\mathcal{V}_2} - \dfrac{1}{\mathcal{V}_1}}{\dfrac{1}{\mathcal{V}_1}} = \mathcal{V}_1\left(\frac{1}{\mathcal{V}_2} - \frac{1}{\mathcal{V}_1}\right) = \mathcal{V}_1\left(\frac{\mathcal{V}_1 - \mathcal{V}_2}{\mathcal{V}_2\mathcal{V}_1}\right) = \frac{\mathcal{V}_1 - \mathcal{V}_2}{\mathcal{V}_2}$$

So

$$\frac{K_2 - K_1}{K_1} = \frac{\mathcal{V}_1 - \mathcal{V}_2}{\mathcal{V}_2}. \tag{12}$$

The equation is *close*, but not exactly equal to the term on the right hand side of Equation (7) what we want. For example, the denominator is $\mathcal{V}_2$, not $\mathcal{V}_1$ as in Equation (7). We need a little more algebra, and the easiest way to do that is to convert Equation (7).

Rewrite Equation (7) as

$$\mathcal{V}_2 - \mathcal{V}_1 = \alpha\mathcal{V}_1 \tag{13}$$

or

$$\mathcal{V}_1 - \mathcal{V}_2 = -\alpha\mathcal{V}_1 \tag{14}$$

which is the numerator for the right hand side of Equation (12) that we want. Solving (13) for $\mathcal{V}_2$ gives

$$\mathcal{V}_2 = \mathcal{V}_1 + \alpha\mathcal{V}_1 = \mathcal{V}_1(1 + \alpha) \tag{15}$$

Finally, substitute Equation (13) and Equation (15) into Equation (12) and simplify to get

$$\frac{K_2 - K_1}{K_1} = \frac{-\alpha\mathcal{V}_1}{\mathcal{V}_1(1 + \alpha)} = \frac{-\alpha}{1 + \alpha}. \tag{16}$$

Or, using Equation (8)

$$\beta = -\frac{\alpha}{1 + \alpha}. \tag{17}$$

Now, we can substitute $\alpha = 0.1$ to get

$$\frac{K_2 - K_1}{K_1} = \beta = -\frac{0.1}{1.1} = -0.090909 \qquad \boxed{\frac{K_2 - K_1}{K_1} = -0.091 \quad \text{or} - 9.1\%} \tag{18}$$

In words, an *increase* in volume of 10% causes a decrease in K of 9.1%.

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Energy Control

1. What should happen in the temperature control algorithm when the measured temperature exceeds the UCL?

   Solution: There is no is no way to cool the water in the tank. When the temperature exceeds the UCL, we simply wait for the water to cool by losing heat to the ambient.

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2. What is the recommended response in the thermal control algorithm when the calculated time for turning on the heater exceeds the deadtime for the salinity control algorithm?

   Solution: When the calculated time for turning on the heater exceeds the deadtime for the salinity control algorithm, the heater-on time should be set equal to the deadtime. This guarantees that the heater does not unnecessarily block to salinity control algorithm from adjusting the salinity.

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3. Flyback diodes are recommended for the solenoids in the salinity control circuit. Why is a flyback diode not needed for the heater in the temperature control circuit?

   Solution: The solenoids have electromagnetic coils that actuate the valve stem. Those coils store energy when power is supplied to open the valve. The flyback diodes provide a path to dissipate the energy stored in the coils in the instant after the power to the coil is turned off..

   The heater is a resistor and does not store energy when power is supplied to it. Therefore, there is no need to provide a path to dissipate energy when the power is turned off on the heater.

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