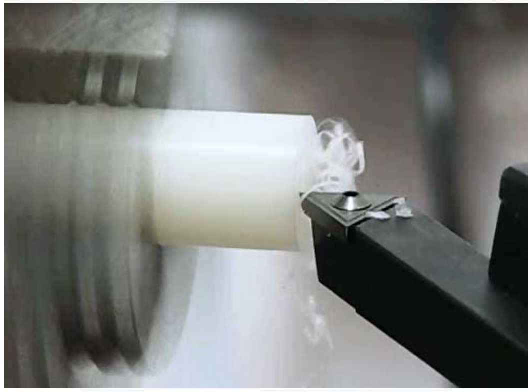
material balance

Engineers design products, and we make them out of different raw materials. We can . . .

- change the shape of a body by adding or removing material
- change the organization of atoms in a material
- combine components to create new substances through chemical reactions

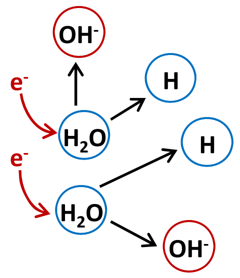
removing material



rearranging atomic structure

Blacksmith hot forging a steel tool using a hammer and an anvil.

chemical reactions



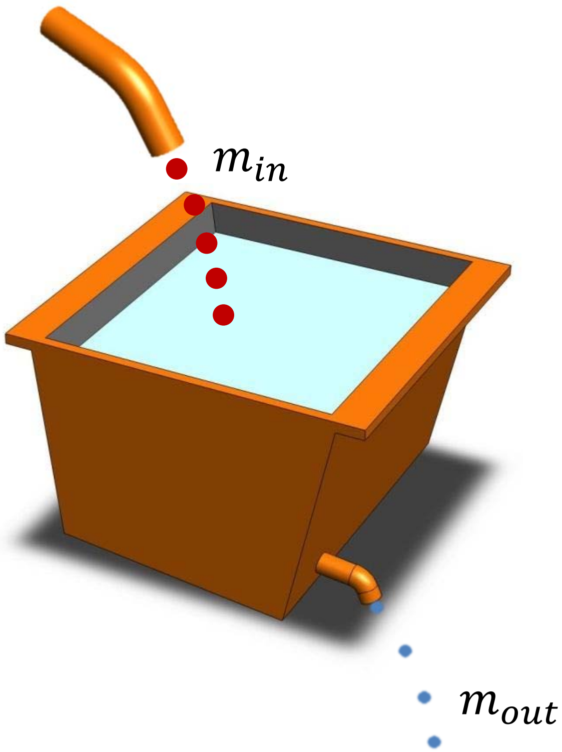
$$2\,H_2O(l) + 2e^- \rightarrow H_2(g) + 2OH^-(aq)$$

we use material balance to analyze all sorts of physical processes (industrial, biological, environmental)

mass can neither be created nor destroyed . . . just rearranged

short of nuclear reactions ($E = mc^2$)

material balance *. . . keeping track of the mass*



$$m_{in} - m_{out} + m_{generated} - m_{consumed} = \Delta m_{system}$$

accumulation of mass in the system

if chemical reactions occur,
new system components may be **generated**
. . . while others are **consumed**

$$2\,H_2O(l) + 2e^- \rightarrow H_2(g) + 2OH^-(aq)$$

$$m_{in} - m_{out} = \Delta m_{system}$$

example applications

- water in a lake
  water-flowing$_{in}$ + rain – water-flowing$_{out}$ - evaporation ≈ Δ water mass

- laundry dryer
  wet-laundry$_{in}$ – partially-dry-laundry$_{out}$ - evaporation = 0

- fishtank project
  NaCl initially in system + NaCl added - NaCl leaving through overflow = Δ NaCl

batch & rate problems

BATCH - start with nothing in system & end with nothing in system

- making a batch of homemade ice cream
- mixing a batch of concrete in a mixer

RATE – continuous flow of inputs and outputs

1. steady state – the amount or type of mass in the system does not change with time
   - a lake at a constant level

     water flowing in + rain – water flowing out – evaporation ≈ **0**

2. non steady state – the amount or type of mass in the system changes with time
   - a lake filling up with water

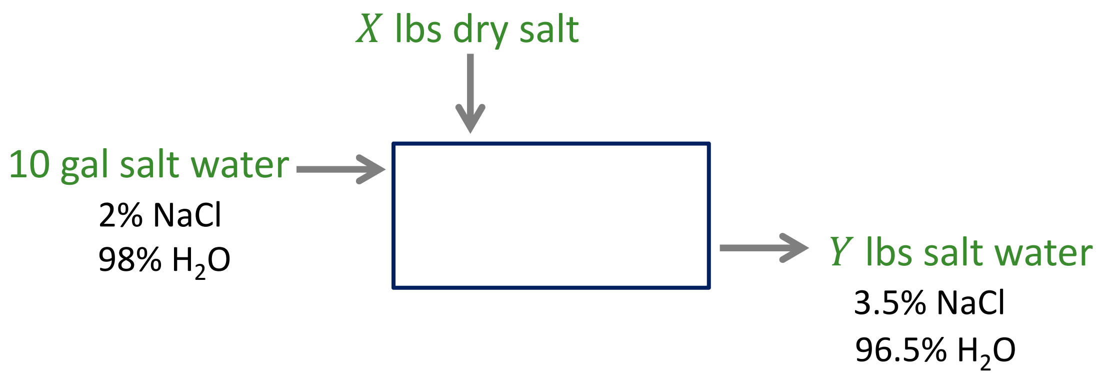
     water flowing in + rain – water flowing out – evaporation ≈ Δ water mass
   - a glacier whose mass is changing

Class Problem

A 10-gallon aquarium contains 2% salt by weight. How much salt would you need to add to bring the salt concentration to 3.5% salt by weight?

1. Draw a diagram to represent the system

2. Label all inputs and outputs, assigning variables to unknowns

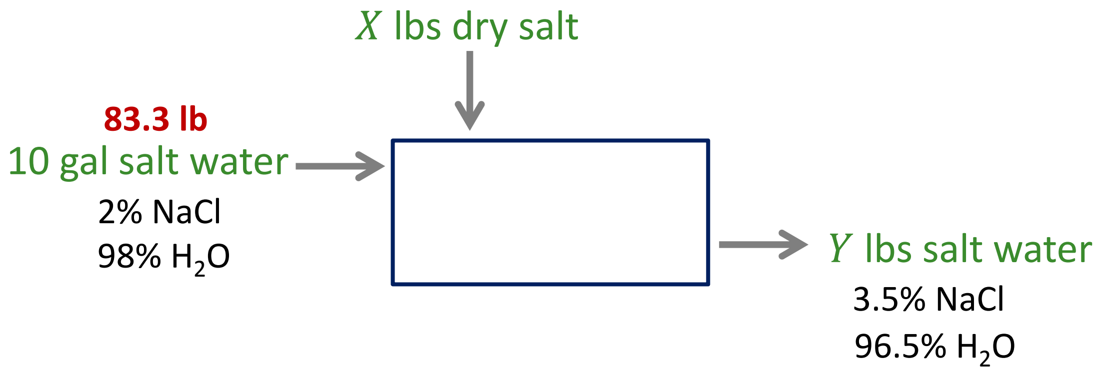


3. Apply conservation of mass to each component (salt & water) and for mixture

first convert the water volume to weight:

$$W_{H_2O} = 10\,gal \cdot \frac{0.1337\,ft^3}{gal} \cdot 62.3\,\frac{lb}{ft^3} = 83.3\;lb$$

3. Apply conservation of mass to each component (salt & water) and for mixture



overall: $m_{in} = m_{out}$

$$83.3\ lb + X = Y$$

(1)

salt: $m_{in} = m_{out}$

$$0.02(83.3\ lb) + X = 0.035 \cdot Y$$

(2)

water: $m_{in} = m_{out}$

$$0.98(83.3\ lb) = 0.965 \cdot Y$$

(3)

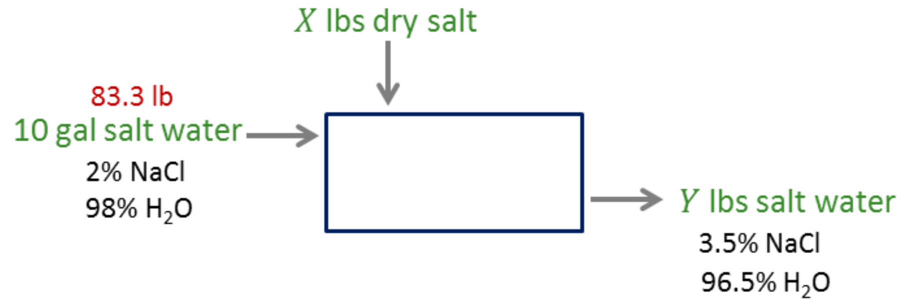
4. Solve for the unknowns (you can use any of the three equations above)

From (3): $Y = \dfrac{0.98\ (83.3\ lb)}{0.965} = 84.6\ lb$

Plug this into (1): $X = Y - 83.3\ lb = 84.6\ lb - 83.3\ lb = \underline{1.29\ lb}$

dry salt to add

Is equation (2) useful??? why?? use it to check your work



problem solving tips

1. Draw a picture of the system. Sometimes it's not easy to determine the boundaries of your system. *(a large river flowing into the ocean for example . . . where does river end and ocean begin?)*

2. Label all inputs and outputs, listing all known quantities & concentrations and assigning variables to the unknowns. This key step is where errors usually occur.

3. Think about the problem a little bit . . . determine if the process is a rate or batch problem. Are components generated or consumed? Revise (1) and (2) if needed.

4. Write conservation of mass (or weight) for each component and for the entire system. Modify the diagram as new information is uncovered.

5. Solve for the unknowns.

6. Reflect on your solution. Do the concentrations or quantities make sense?

WARNING avoid trying to just solve these problems in your head . . . use the systematic approach above