Energy Balances

An energy balances expresses the conservation principle for the energy content of a system.

Energy Storage in a Fluid

Consider a tank of water with an immersed electric resistance heater as depicted in Figure 1. Applying the First Law of Thermodynamics (the energy conservation principle) gives

$$\Delta E = E_{\text{in}} - E_{\text{out}} \tag{1}$$

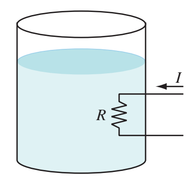


Figure 1: Electric resistance heater in a tank of water.

where ΔE is the change in energy content of water during a specified time interval Δt, E_{in} is the energy added to the water during Δt, and E_{out} is the energy lost to the ambient during Δt.

Since the tank is open to the atmosphere, the additional of energy will only cause an increase in temperature, not pressure. The change in energy of the tank is

$$\Delta E = mc_p \Delta T \tag{2}$$

where m is the mass, c_p is the specific heat at constant pressure, and ΔT is the change in temperature.

Review: Power Dissipated by Direct Elecrical Current

The power dissipated as electrical current, I, flows through a resistor, R, is

$$P = VI = I^2 R = \frac{V^2}{R}$$

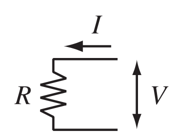


Figure 2: Current flow through a resistor.

where V is the voltage drop across the resistor.

You only need to remember one of those formulas. The others can be derived by applying Ohm's law

$$V = IR \tag{3}$$

For example,

$$V = IR \Longrightarrow I = \frac{V}{R} \qquad \text{therefore} \qquad P = I^2 R = \left(\frac{V}{R}\right)^2 R = \frac{V^2}{R}$$

Energy Input from the Heater

For the electric resistance heater shown in Figure 1, the energy input during Δt is

$$E_{\text{in}} = VI\Delta t \tag{4}$$

where V is the voltage across the resistor, I is the current through the resistor, and VI is the *rate* at which electrical power is dissipated. Substituting Equation (2) and Equation (4) into Equation (1), and neglecting heat loss to the ambient (such that $E_{\text{out}} = 0$) gives

$$\Delta E = E_{\text{in}} \quad \Longrightarrow \quad mc_p\Delta T = VI\Delta t \quad \Longrightarrow \quad \Delta T = \frac{VI\Delta t}{mc_p} \tag{5}$$

The last equation on the right is the temperature rise that would occur during time Δt when the electrical heater is operating.

Often the volume of fluid is given (or to be computed). The volume $\mathcal{V}$ and mass m are related by the density, ρ

$$m = \rho\mathcal{V}. \tag{6}$$

A Note on Units: The units of c_p are usually written J/(kg K), but the temperature change is usually expressed in °C. For example, consider the increase in energy content when 2 kg of water has its temperature increase by 2 °C.

$$\Delta E = (2\,\text{kg})\left(4180\,\frac{\text{J}}{\text{kg}\,\text{K}}\right)(2\,^\circ\text{C}) = 16,720\ \frac{\text{kg}}{}\frac{\text{J}}{\text{kg K}}\frac{^\circ\text{C}}{} = 16,720\ \text{J}$$

In this situation, the units of °C in the numerator cancel the units of K in the numerator. Why? The energy storage occurs because there is a *change* in temperature, and when temperature changes are involved, Δ°C and ΔK are interchangeable. For example, suppose the temperature of the water changes from 21 °C to 23 °C.

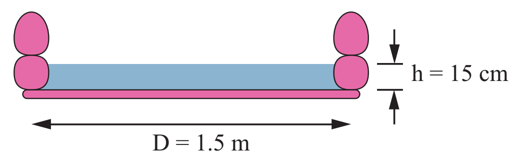
$$^\circ\text{C units:} \quad T_2 - T_1 = 23\,^\circ\text{C} - 21\,^\circ\text{C} = 2\,^\circ\text{C change}$$

$$\begin{aligned}\text{K units:} \quad T_2 - T_1 &= (23 + 273.15)\,\text{K} - (21 + 273.15)\,\text{K} \\ &= (23 - 21) + (273.15 - 273.15)\,\text{K} = 2\,\text{K change}\end{aligned}$$

It does not hurt to work with temperature differences in K, but as long as only temperature *differences* matter, working with °C is also correct because the offset of 273.15 °C cancels.

Example Problems

1. How long would it take to raise the temperature of a small, inflatable swimming pool 15 cm deep and 1.5 m in diameter by 1 °Cusing an electrical heater driven by a 12 VDC power supply capable of providing 10 A? What is the electrical resistance of the heater required?



2. Re-compute the preceding problem with a 120V AC power supply. For an AC voltage, the power is $P = V_{\mathrm{rms}} I$ where $V_{\mathrm{rms}} = V_{\mathrm{peak}}/\sqrt{2}$.