Problem Statement:

If the saltwater in the fish tank is 0.05 wt % NaCl, what is the mass fraction of the water after 1 cm^3 of saltwater with 2.0 wt % NaCl is added? Assume that the water is at 20°C and that the density of saltwater can be estimated as

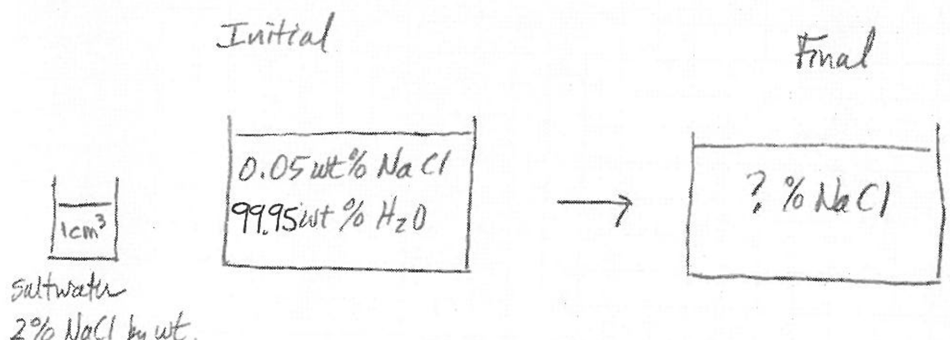
$$\rho = 998.208 + 714.29\, X_s \quad \frac{kg}{m^3}$$

where X_s is the mass fraction of NaCl. Assume that the fish tank has an inside diameter of 1.6 inch and that the water depth is 1.5 inches.

Solution: Follow the steps listed in the lecture notes

1. There are no flows, and there are well-defined initial and final states. Therefore, this is a batch process.

2. Schematic



Knowns:

Initial volume of the tank: $V_i = \frac{\pi}{4} d^2 h = \frac{\pi}{4}\left(1.6\,in \times \frac{2.54\,cm}{in}\right)^2 \left(1.5\,in \times \frac{2.54\,cm}{in}\right)$

$$= 49.42\ cm^3$$

Initial mass in the tank: $m_i = \rho V_i$

$$m_i = \left[998.208 + (714.29)\left(\frac{0.05}{100}\right)\frac{kg}{m^3}\right](49.42\ cm^3)\left(\frac{1\,m}{100\,cm}\right)^3$$

$m_i = 4.935 \times 10^{-2}$ kg $\qquad m_i = 49.35$ g

3. Write down masses of species and total mass at initial and final conditions, Let "add" be subscript for the 1 cm^3 of water added.

	Initial	Final
salt:	$m_{add} X_{s,add} + m_i X_{s,i}$	$m_f X_{s,f}$
water:	$m_{add} X_{w,add} + m_i X_{w,i}$	$m_f X_{w,f}$

Total: $m_{add}(X_{s,add} + X_{w,add}) + m_i(X_{s,i} + X_{w,i})$

$= m_{add} + m_i$

because $X_{s,add} + X_{w,add} = 1$

$X_{s,i} + X_{w,i} = 1$

from the definition of mass fraction

Final total: $m_f(X_{s,f} + X_{w,f}) = m_f$

because $X_{s,f} + X_{w,f} = 1$

4. Set initial mass equal to final mass for each species and for totals

salt: $m_{add} X_{s,add} + m_i X_{s,i} = m_f X_{s,f}$ (1)

water $m_{add} X_{w,add} + m_i X_{w,i} = m_f X_{w,f}$ (2)

total $m_{add} + m_i = m_f$ (3)

5. In these equations, only m_f, $X_{s,f}$ and $X_{w,f}$ are unknown

We can directly compute $m_{add} = \rho V_{add}$

With m_{add} and m_i known, compute m_f from equation (3)

With m_f known, compute $X_{s,f}$ from equation (1)

Use equation (2) as a check on other calculations

$$m_{add} = \rho \, V_{add} \qquad \text{Given: } V_{add} = 1\,cm^3 \qquad X_{s,add} = 0.02$$

$$= \left[998.208 + (714.29)(0.02)\frac{kg}{s}\right](1\,cm^3)\left(\frac{1\,m}{100\,cm}\right)^3$$

$$= 1.0125 \times 10^{-3}\,kg$$

$$m_{add} = 1.0125\,g$$

Note: we carry extra digits through intermediate calculations and then round at the end.

Equation (3) $\Rightarrow$ $m_f = m_{add} + m_i = 1.0125g + 49.35g$

$$m_f = 50.36\,g$$

Equation (1) $\Rightarrow$ $X_{s,f} = \frac{1}{m_f}\left(m_{add} X_{s,add} + m_i X_{s,i}\right)$

$$= \frac{1}{50.36g}\left[(1.0125g)(0.02) + (49.35g)(0.0005)\right]$$

$$X_{s,f} = 8.9208 \times 10^{-4} = 0.00089 = 0.089\%$$

$$\boxed{X_{s,f} = 0.089 \text{ wt\% NaCl}}$$

CHECK:

From the definition of mass fraction: $X_{w,f} + X_{s,f} = 1$

$$\Rightarrow X_{w,f} = 1 - X_{s,f} = 1 - 0.0008921$$

$$= 0.999108$$

Substitute into Equation (2)

$$(1.0125g)(0.98) + (49.35g)(0.9995) \stackrel{?}{=} (50.36g)(0.999108)$$

$$50.318 \stackrel{?}{=} 50.315g \checkmark$$

Difference is within rounding error