ME 121: Mass Balance Problems Gerald Recktenwald
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# 1 Goal: An Analytical Model of Salinity Balance

These notes introduce methods to analyze mixing of batches and flows of material. Mixing problems arise in applications of chemical, civil and mechanical engineering. For the fish tank problem, these analytical tools enable us to predict the response of the fish tank to additions of salty and fresh water.

## 1.1 Learning Objectives

After studying these notes you should be able to

1. Identify whether a process is batch, steady flow or unsteady flow;
2. Draw a schematic of a steady flow process given a verbal description of inputs and outputs;
3. Write the mass balance equations for batch and steady flow processes; and
4. Use a systematic procedure to solve batch and steady-flow mass balance problems.

These notes are written with the assumption that you already understand

- how to work with units in engineering analysis;
- how to compute mass fractions; and
- how to use algebra to solve for unknowns in a system of two equations.

## 1.2 Other references

Engineering models of mass balance appear in books on Thermodynamics and Chemical Engineering processes. An excellent free reference is the Wikibook, *Introduction to Chemical Engineering Processes*[1]

# 2 Types of Mass Balance Problems

We begin by defining terms used to analyze mixtures. We then describe batch mixing, steady flow mixing, and unsteady mixing problems.

Most of the salinity modeling for the fish tank can be described by a batch model even though the reality is more complicated. It is important to recognize when a batch model applies. Like all models used in engineering, we should not apply the batch model to problems when it is not appropriate. Therefore, we introduce the steady flow and unsteady models as references.

[1] The whole book is found at `http://en.wikibooks.org/wiki/Introduction_to_Chemical_Engineering_Processes`. A helpful starting point for the material discussed in these notes is the section *What is a mass balance?*, `http://en.wikibooks.org/wiki/Introduction_to_Chemical_Engineering_Processes/What_is_a_mass_balance%3F`.

## 2.1 Common Themes and Definitions

Different mass balance problems share the same core idea that *mass is always conserved*. We will show three different types of problems: batch, steady flow and unsteady flow. The difference is in how the engineering analysis is performed.

Another common idea is that all problems involve a mixing of ingredients. We need to be able to quantitatively define the constituents of each ingredient.

**Mole:** A mole is $6.022 \times 10^{23}$ unitary entities of a substance. The unitary entities could be molecules, atoms, ions, electrons or others. In other words, a mole is a number or count of individual things. The number of unitary entities in a mole is called *Avogadro's number.*

$$\mathcal{N}_A = 6.022 \times 10^{23}$$

Avogadro's number has no units since it is simply the number of things.

**Mass Fraction:** Mass fraction is the mass ratio of a constituent substance to the total mass of a mixture of constituents. The symbol for mass fraction is capital $X$. The constituent is identified by a subscript. Thus, the mass fraction of constituent $A$ in a mixture is

$$X_A = \frac{\text{mass of } A}{\text{mass of the mixture}}$$

The total mass of a mixture is the sum of the masses of its constituents,

$$m_A + m_B + m_C + \cdots = m_{\text{mixture}} \tag{1}$$

where $m_A$ is the mass of constituent A, and B, C, etc. are other constituents of the mixture. Dividing both sides of Equation (1) gives must add to one.

$$X_A + X_B + X_C + \cdots = 1 \tag{2}$$

This equation applies to any mixture. For the fish tank model, the only components are salt (NaCl) and pure water[2]. Therefore, applying Equation (2) to the fish tank gives

$$X_{\text{salt}} + X_{\text{water}} = 1 \tag{3}$$

**Weight Fraction:** Sometimes concentrations are expressed as weight fractions, which are numerically equivalent to mass fractions

$$\text{Weight fraction} = \frac{\text{weight of } A}{\text{weight of the mixture}} = \frac{\text{mass of } A \times g}{\text{mass of the mixture} \times g} = \frac{\text{mass of } A}{\text{mass of the mixture}}$$

As with mass fractions, the sum of all weight fractions in a mixture must equal 1.0.

**Weight Percent:** In our mathematical analysis, we use the mass fraction, $X_s$, to indicate the amount of salt in the mixture. However, since $X_s$ is small in our fish tank, it is also convenient to use weight percent salt, which is $100X_s$. The advantage is simply that weight percent is a larger number.

When we need a symbol for weight percent, we will use $S$.

$$S = 100X_s \qquad \text{weight percent salt} \tag{4}$$

[2]Real fish tanks, and seawater are much more complex mixtures.

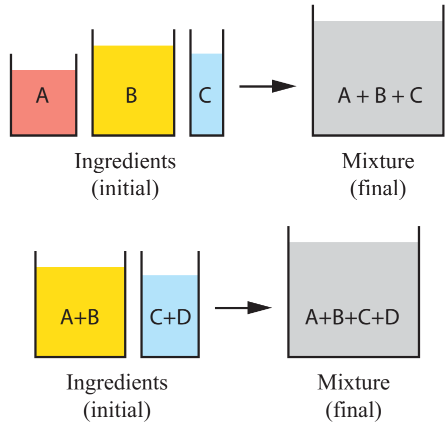



**Figure 1:** Schematic of batch mixing processes. Constituents A, B, C and D may or may not undergo chemical interactions.

**Example 2.1 Mass fraction in a salt water mixture**

A sample of saltwater mixture is labeled 0.05 wt % salt. What are the mass fractions of salt and water in this mixture?

From the definition of mass fraction and weight percent,

$$X_s = \frac{S}{100}$$

and we are given $S = 0.05\,\%$, so that

$$X_s = \frac{1}{100} \times 0.05\,\text{wt.}\,\% \ \text{ salt} = 0.0005$$

Remember that $X$ is a ratio, and therefore is dimensionless. Applying Equation (3) gives

$$X_w = 1 - X_s = 1 - 0.0005 = 0.9995$$

□

## 2.2 Batch Processes

A batch process is a simple model of mixing that can be described in terms of distinct initial and final states. Figure 1 shows schematic representations of batch systems. Batch processes have these characteristics:

- In batch processes, there is a *initial* state, typically with separate constituents, and an *final* state, where the constituents are mixed.
- In batch processes, there is no flow of materials.
- Time is not a factor. In other words, although the change from initial to final state implies a difference in time, the rate at which the mixing happens is not a factor and end states themselves are considered to be unchanging in time.

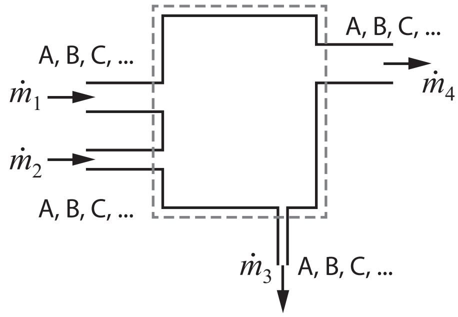



**Figure 2:** Steady flow model of a mixing process.

If it helps in the analysis, we can conceptually reverse the order of the process such that the initial state has mixed components and the final state has unmixed components. For salt and water mixtures, un-mixing will not happen spontaneously[3], so we should think of un-mixing only if it helps to set up a problem for analysis. Sometimes, problems with steady flows can be converted to batch problems. However, if there is no flow, then use a batch model in your analysis.

## 2.3 Steady flow Problems

Steady flow systems can be identified by the following characteristics.

1. The system has continuous flow rates across its boundaries. Contrast with a batch process.
2. The flow rates do not change with time. Contrast with an unsteady process.
3. The generation and/or consumption of species does not change with time. Contrast with an unsteady process.

Figure 2 is a schematic of a steady flow system. The dashed line provides an imaginary boundary that separates the system, which is inside the boundary, from the environment. Usually we refer to the system boundary as a *control volume.* In this context, "control" simply identifies a region in which overall mass, species and/or energy balances are performed.

In Figure 2 there are two incoming streams and two outgoing streams. In general, there is no limit on the number of streams, but in typical systems the total number of streams is small, say 2 to 5. The total mass flow rate in each stream is designated by $\dot{m}$ and a subscript to identify the stream.

In a steady flow problem, the flow rates are steady, i.e., they do not change with time. As a consequence, the mass in the control volume is also steady. Furthermore, we assume that the composition of the mixture is uniform inside the control volume. In fact, the concentrations of constituents immediately downstream of the inlets will be close to the constituents in the corresponding inlet stream. However, as long as the control volume is large enough, we will assume that the streams leaving the control volume have a composition determined by a complete mixture of the inlets. In other words, with reference to Figure 2, the composition of stream 3 and stream 4 will be the same, and will be determined by the flow rates and compositions of the entering streams 1 and 2.

Steady flow problems involve flow *rates*, meaning that the material is continuously crossing the boundary of the control volume. There is no *initial* state and *final* state because the system experiences a continuous process. If we were to attach sensors to the input and output streams (say

[3] And not without additional work input from an external source.

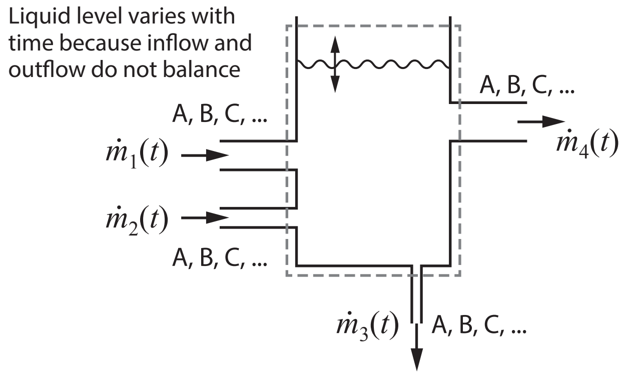



**Figure 3:** Unsteady flow model of a mixing process.

temperature or pressure or salinity sensors), these sensors would have different values depending on their location in the streams, but the output from the sensors would not vary in time because the entire system is steady.

## 2.4 Unsteady Flow Problems

Figure 3 is a schematic representation of an unsteady flow and mixing problem. In an unsteady flow problem the streams entering and leaving the control volume can change with time. The material in the mixing region can also accumulate (or decrease) with time, which would cause a change in level of the tank in Figure 3. Unsteady flow problems are more complex than steady flow problems. We will stick to batch and steady flow problems in this course.

## 2.5 Flow Rate Definitions

Mass flow rate is a quantity indicating the amount of mass crossing a defined boundary per unit time. The symbol for mass flow rate is $\dot{m}$,

$$\dot{m} = \frac{\text{mass crossing a boundary}}{\text{period of time}}.$$

The amount of material crossing a boundary can also be expressed as a volumetric flow rate. The volumetric flow rate is designated $Q$,

$$Q = \frac{\text{volume crossing a boundary}}{\text{period of time}}.$$

Note that the symbol for volumetric flow rate, $Q$, is also used in heat transfer to denote a rate of heat flow[4].

We can get a physical feel for volumetric and mass flow rates by considering the filling of a bucket from a faucet as depicted in Figure 4. If the flow rate is steady, then

$$Q = \frac{\Delta V}{t_2 - t_1} = \frac{\Delta V}{\Delta t} \tag{5}$$

[4]There are only 26 letters in the roman alphabet, so some limits on the choice of symbols is inevitable. Within sub-disciplines the symbols are consistent. Across sub-disciplines the symbols get reused.

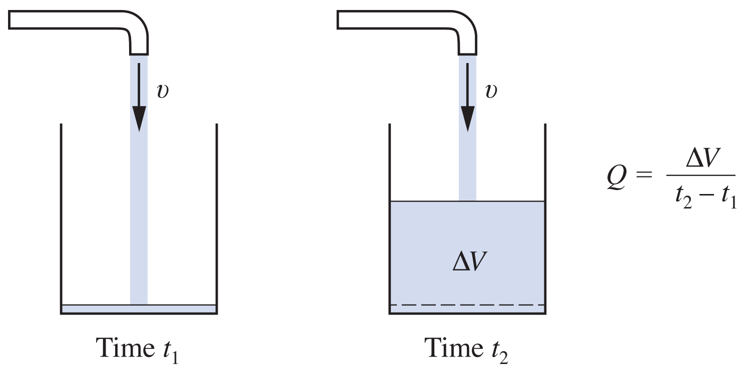



**Figure 4:** Volumetric flow rate during the filling of a bucket. The flow rate is assumed to be steady.

where $\Delta V$ is the change in volume during the time interval $t_2 - t_1 = \Delta t$, and $v$ is the downward fluid velocity.

The mass flow rate could be measured if a weighing scale was placed under the bucket in Figure 4. We would then measure the change in mass of water

$$\dot{m} = \frac{\Delta m}{t_2 - t_1} = \frac{\Delta m}{\Delta t} \tag{6}$$

where $\Delta m$ is the change in mass during the time interval $\Delta t$.

In practical engineering problems we cannot usually measure flow rates with buckets. One of the primary goals of using mass balances is to reduce the number of measurements by relating flow rates to each other. We also use fluid property definitions. Mass and volume are related by the density

$$\rho = \frac{\text{mass of material}}{\text{volume of material}} \tag{7}$$

For many common liquids and gases, the density is tabulated or available in formulas called equations of state. For our purposes, we will assume that the density is known.

From the definition of density in Equation (7), the relationship between mass and volume, as well as the relationship between mass flow rate and volumetric flow rate is

$$\begin{array}{cccc} \text{mass} & = \text{density} & \times & \text{volume} \\ m & = \rho & & V \\ & & & \\ \begin{array}{c}\text{mass}\\ \text{flow rate}\end{array} & = \text{density} & \times & \begin{array}{c}\text{volumetric}\\ \text{flow rate}\end{array} \\ \dot{m} & = \rho & & Q \end{array}$$

where $V$ is the volume of the sample having mass $m$. Note that the flow rate can be steady, or it can change with time.

When considering flow in pipes, it is very helpful to relate the flow rates to the velocity of the fluid in the pipe. Figure 5 depicts the flow of water from a round pipe into a bucket. In a short time interval $\Delta t$, a slug of water in the pipe moves a distance $L = v\Delta t$, where $v$ is the velocity of water in the pipe. The volume of water in that slug is

$$\Delta V = LA = (v\Delta t)A$$

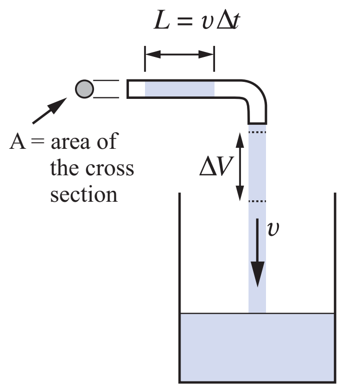



**Figure 5:** Volumetric flow rate related to the velocity of flow in a pipe.

where $A$ is the cross-sectional area of the duct. Think of $LA$ as the volume swept during $\Delta t$ by the slug of water moving at velocity $v$. Dividing both sides of the preceding equation by $\Delta t$ gives

$$\boxed{Q = vA} \tag{8}$$

and since $\dot{m} = \rho Q$, multiplying both sides of Equation (8) by $\rho$ gives

$$\boxed{\dot{m} = \rho v A} \tag{9}$$

Equation (8) and (9) are two fundamental equations for the volumetric and mass flow rate through a pipe or duct.

#### 2.5.1 Flow Rates of Constituents

In the preceding section, the fluid velocity, mass flow rate and volumetric flow rate were not linked to individual constituents. In general, when no reference is made to mixture components, the flow rates and velocities are assumed to be those of the *mixture*.

To account for mass flow of a single consituent, simply multiply the mixture flow rate by the mass fraction of the component. For example,

$$\dot{m}_A = \dot{m} X_A. \tag{10}$$

#### 2.5.2 Mass Balance Equation for the System

The *Conservation Principle* is that the physical properties of material in a system is completely accounted for by three processes: (1) the net inflow or outflow of material across the system boundaries, (2) the generation or consumption of material in the system by chemical reactions, or (3) a change in the storage of material in the system. By *material* we can mean either the total mass in the system or individual chemical species. The conservation principle can be expressed in the following quasi-mathematical expression.

$$\begin{matrix}\text{Rate of} \\ \text{accumulation}\end{matrix} = \begin{matrix}\text{Rate of} \\ \text{inflow}\end{matrix} - \begin{matrix}\text{Rate of} \\ \text{outflow}\end{matrix} + \begin{matrix}\text{Rate of} \\ \text{generation}\end{matrix} - \begin{matrix}\text{Rate of} \\ \text{consumption}\end{matrix} \tag{11}$$

This generic equation will be applied to the conservation of overall mass, and to the conservation of individual chemical species.

**Overall Mass Balance** Mass is not created or destroyed by chemical reactions. Therefore, applying the conservation principle in Equation (11) to the mass in a system gives

$$\begin{matrix}\text{Rate of} \\ \text{accumulation} \\ \text{of mass}\end{matrix} = \begin{matrix}\text{Rate of} \\ \text{inflow} \\ \text{of mass}\end{matrix} - \begin{matrix}\text{Rate of} \\ \text{outflow} \\ \text{of mass}\end{matrix} . \tag{12}$$

Equation (12) *always* applies. We need to be more specific about each of the terms before this equation is useful in an engineering analysis. In most situations, this general formula reduces to a much simpler form.

**Mass Balance for constituents** Chemical reactions generally involve the combination of one set of chemical species (e.g., molecules, atoms, ions, etc.) to form other set of chemical species. For a general, unsteady system as depicted in Figure 3, the conservation of species $k$ requires

$$\begin{matrix}\text{Rate of} \\ \text{accumulation} \\ \text{of } k\end{matrix} = \begin{matrix}\text{Rate of} \\ \text{inflow} \\ \text{of } k\end{matrix} - \begin{matrix}\text{Rate of} \\ \text{outflow} \\ \text{of } k\end{matrix} + \begin{matrix}\text{Rate of} \\ \text{generation} \\ \text{of } k\end{matrix} - \begin{matrix}\text{Rate of} \\ \text{consumption} \\ \text{of } k\end{matrix} \tag{13}$$

Equation (13) *always* applies, but it is not particularly useful in this form except as a conceptual statement about the processes that contribute to conservation of species.

# 3 Analysis of Mass Balance Problems

We now apply the basic definitions and conservation principles to methods for analyzing practical problems.

## 3.1 Analysis of Batch Problems

Batch problems have an identifiable initial and final state. Batch problems do not involve flow. Since there is no inflow or outflow, the rate of accumulation (or loss) of mass inside the system must be zero. Therefore, for batch problems the mass conservation principle can be reduced to

$$\text{Initial mass} = \text{Final mass} \tag{14}$$

To use this principle to set up and solve problems, we need to translate the words into a form that can be used in equations. We use $m$ to denote mass, and A, B, C, etc. to identify constituents. The mass conservation principle applies to the total mass in the system, and to each of the constituents.

$$\begin{aligned} m_{\text{initial}} &= m_{\text{final}} && \text{total mass} \\ m_{A,\text{initial}} &= m_{A,\text{final}} && \text{constituent A} \\ m_{B,\text{initial}} &= m_{B,\text{final}} && \text{constituent B} \\ m_{C,\text{initial}} &= m_{C,\text{final}} && \text{constituent C} \\ &\vdots \end{aligned} \tag{15}$$

It is clumsy to write “initial” and “final” in mathematical expressions, so we will use “i” for initial and “f” for final. We also use the convention that if no constituent (A,B,C,. . . ) is identified, then the mass is the total system mass. Thus, the mass balance principle for batch process can be written.

$$\begin{aligned} m_i &= m_f \\ m_{A,i} &= m_{A,f} \\ m_{B,i} &= m_{B,f} \\ m_{C,i} &= m_{C,f} \\ &\vdots \end{aligned} \tag{16}$$

**Stepwise Procedure for Batch Problems**

The following steps are used to solve a batch problems. Of course, you must first identify whether the problem is suitable for a batch model. If yes, procede with the batch analysis. If not, use a steady flow or unsteady flow model, as appropriate.

1. Make a sketch of the system, and identify the initial state and the final state.
2. Set masses at initial state equal to masses at the final state, i.e., invoke the mass conservation principle, as in Equation (16). This step requires identification of the total mass, and the mass of each constituent for both the initial and final states.
3. Identify which of the quantities of mass and species are known and which are unknown.
4. Use algebra to solve for the unknowns.
5. Check your work for consistency.

The stepwise procedure may seem confining or pedantic. However, as mixing problems become more complex, the systematic approach provides an organized approach. Stepwise procedures help solve problems that may seem impossible at first.

Regardless of the procedure, **Always check your units!**

### Example 3.1 Adjusting Salinity of a Saltwater Mixture

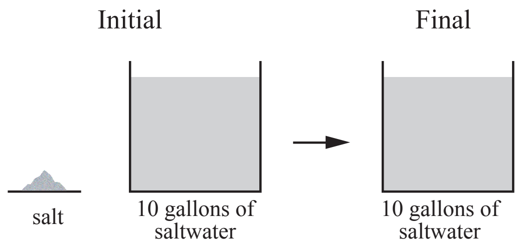
**Problem Statement:**

> A 10-gallon aquarium contains 2 percent salt by weight. How much salt would you need to add to bring the salt concentration to 3.5 percent salt by weight?

Before starting the analysis, we should determine whether this is a batch, steady flow or unsteady flow problem. Since we are given a known initial state and final state, and there is no flow, we can treat this as a batch problem.

| | |
|---|---|
| Initial: | A mixture with known volume and mass fraction of salt. |
| Final: | Mixture with desired (known) mass fraction of salt and unknown total mass of the mixture. |

**Step 1: Make a sketch, and identify initial and final state.**




**Note:** The total volume of salt added is small. As with most problems involving small changes in salinity, we neglect any change in volume of the fluid. Therefore, the tank is assumed to have 10 gallons of mixture before *and after* the salt is added. We are making an *assumption about volume*, not and assumption about mass.

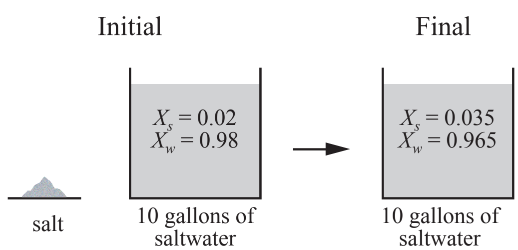
Use these symbols:

| | |
|---|---|
| $m_s$ | mass of salt |
| $m_{\text{mix}}$ | mass of saltwater mixture |
| $X_s$ | mass fraction of salt |
| $X_w$ | mass fraction of water |

We know the initial and final mass fractions of the salt and water

| | | |
|---|---|---|
| Initial: | $X_s = 0.02$ | given |
| | $X_w = 0.98$ | because $X_w = 1 - X_s$ |
| Final: | $X_s = 0.035$ | given |
| | $X_w = 0.965$ | because $X_w = 1 - X_s$ |

We can re-draw the system by adding labels to the water mixtures. Note that you don't need to re-draw the system on a homework solution. You can simply add the mass fractions (or other labels) to the original drawing.




We also know the following physical parameters and conversion factors.

$$1 \text{ gallon} = 0.1337\,\text{ft}^3, \qquad \text{density of water} = \rho = 62.3\,\frac{\text{lb}_\text{m}}{\text{ft}^3}$$

From this we can compute the mass of the water that occupies 10 gallons

$$m_w = \rho V = 62.3\,\frac{\text{lb}_\text{m}}{\text{ft}^3} \times 10\,\text{gal} \times \frac{0.1337\,\text{ft}^3}{1\,\text{gal}} = 83.3\,\text{lb}_\text{m} \tag{17}$$

Remember that the mass of water in all initial ingredients must equal the mass of water in the final mixture. Since there is only one fluid volume in both the initial and final states, $m_w$ is both the initial mass of water and the final mass of the water.

**Step 2: Set masses equal for initial and final state.**
Using symbols for all quantities, we can write down the masses of the components before and after the mixing, and set them equal.

$$\text{salt:} \quad m_{s,\text{add}} + X_{s,i} m_{\text{mix},i} = X_{s,f} m_{\text{mix},f} \tag{18}$$

$$\text{water:} \quad X_{w,i} m_{\text{mix},i} = X_{w,f} m_{\text{mix},f} \tag{19}$$

$$\text{total:} \quad m_{\text{mix},i} + m_{s,\text{add}} = m_{\text{mix},f} \tag{20}$$

where $m_{s,\text{add}}$ is the unknown amount of salt to be added, and $m_{\text{mix}}$ is the mass of the saltwater mixture. This set of terms looks intimidating, but the mass fractions are all known. There are really just two unknowns, $m_{s,\text{add}}$ and $m_{\text{mix},f}$.

**Step 3: Identify knowns and unknowns**
These equations will look less intimidating when we substitute known values. First we calculate the initial mass of the mixture using

$$m_w = X_{w,i} m_{\text{mix},i}$$

where $m_w$ is from Equation (17). Therefore,

$$m_{\text{mix},i} = \frac{m_w}{X_{w,i}} = \frac{83.3\,\text{lb}_\text{m}}{0.98} = 85.0\,\text{lb}_\text{m}$$

Substituting the preceding value of $m_{\text{mix},i}$ and other knowns into Equation (18) through Equation (20) gives

$$m_{s,\text{add}} + (0.02)\,(85.0\,\text{lb}_\text{m}) = (0.035)\,m_{\text{mix},f} \tag{21}$$

$$(0.98)\,(85.0\,\text{lb}_\text{m}) = (0.965)\,m_{\text{mix},f} \tag{22}$$

$$85.0\,\text{lb}_\text{m} + m_{s,\text{add}} = m_{\text{mix},f} \tag{23}$$

This is a set of three equations in the two unknowns $m_{s,\text{add}}$ and $m_{\text{mix},f}$. In the next step we will work with two of those three equations, and use the third to check our algebra.

**Step 4: Use algebra to solve for the unknowns**
Solve Equation (22) for $m_{\text{mix},f}$

$$\begin{aligned}
(0.98)\,(85.0\,\text{lb}_\text{m}) &= (0.965)\,m_{\text{mix},f} \\
83.30\,\text{lb}_\text{m} &= (0.965)\,m_{\text{mix},f} \\
m_{\text{mix},f} &= 86.32\,\text{lb}_\text{m} \\
\boxed{m_{\text{mix},f} = 86.32\,\text{lb}_\text{m}}
\end{aligned} \tag{24}$$

Now that $m_{\text{mix},f}$ is known, use either Equation (21) or Equation (23) to solve for $m_{s,\text{add}}$. We will use Equation (21) to solve for $m_{s,\text{add}}$, and save Equation (23) for a final check.

$$\begin{aligned}
m_{s,\text{add}} + (0.02)\,(85.0\,\text{lb}_\text{m}) &= (0.035)\,m_{\text{mix},f} \\
m_{s,\text{add}} + 1.700\,\text{lb}_\text{m} &= (0.035)\,(86.32\,\text{lb}_\text{m}) \\
m_{s,\text{add}} &= 3.021\,\text{lb}_\text{m} - 1.700\,\text{lb}_\text{m} \\
m_{s,\text{add}} &= 1.321,\text{lb}_\text{m} \\
\boxed{m_{s,\text{add}} = 1.32\,\text{lb}_\text{m}}
\end{aligned} \tag{25}$$

**Step 5: Check for consistency**
Use Equation (23) to check the calculations

$$85.0\,\text{lb}_\text{m} + m_{s,\text{add}} = m_{\text{mix},f}$$
$$85.0\,\text{lb}_\text{m} + 1.32\,\text{lb}_\text{m} \stackrel{?}{=} 86.32\,\text{lb}_\text{m}$$
$$86.32\,\text{lb}_\text{m} = 86.32\,\text{lb}_\text{m}\ \checkmark$$

Therefore, the solution to Equation (21) and Equation (22) also satisfies Equation (23), as it must. □

## 3.2 Analysis of Steady Flow Problems

For steady flow, there can be no accumulation or loss of mass inside the system. The mass conservation principle reduces to

$$\begin{matrix}\text{Rate of} \\ \text{inflow of mass}\end{matrix} = \begin{matrix}\text{Rate of} \\ \text{outflow of mass}\end{matrix} \tag{26}$$

In situations where there are several inflow and outflow boundaries, Equation (26) is written with a summation notation

$$\sum_{\text{in}} \dot{m}_i = \sum_{\text{out}} \dot{m}_i \tag{27}$$

where the index $i$ is interpreted as listing the numerical subscripts associated with either the inlet or outlet boundaries. Since the problem is steady, there is no subscript "i" for initial or subscript "f" for final.

The direction of mass flow rates across the system boundary is significant. Assuming that the direction of the arrows in Figure 2 are consistent with the actual flow directions across the boundary, the overall mass balance for the system depicted in Figure 2 is

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3 + \dot{m}_4. \tag{28}$$

Equation (26), Equation (27) and Equation (28) all express the mass balance for the system depicted in Figure 2. The direction of the arrows associated with the flow rates in Figure 2 determine whether the individual terms are placed on the left (inflow) or right (outflow) sides of Equation (28).

Mass flows crossing the boundaries can carry a different composition of constituents. Therefore, in addition to a mass balance equation for the system, we need to write *species balance* equations (just as we did for the batch model) that express the balance in the rates of inflow and outflow of each species across the system boundary. The rate at which species A crosses boundary 1 is

$$\dot{m}_1 X_A$$

which has units of $\text{kg}_A/\text{s}$ in SI units. Remember that the mass fraction $X_A$ is dimensionless. Applying the conservation principle to species A for the system depicted in Figure 2 gives

$$\dot{m}_1 X_{A,1} + \dot{m}_2 X_{A,2} = \dot{m}_3 X_{A,3} + \dot{m}_4 X_{A,4}$$

In general, the values of $X_{A,1}$, $X_{A,2}$, $X_{A,3}$ and $X_{A,4}$ will be not be equal. Analogous balance equations applied to species B and C.

**Example 3.2 Steady flow balance equations for system in Figure 2**

What are the mass balance equations for the system depicted in Figure 2?

The overall mass balance for the mixtures enterring and leaving the system are,

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3 + \dot{m}_4.$$

Mass is also conserved for each of the species. At each boundary, the flow of a species is equal to the mixture flow rate times the species concentration. Thus, for example, the flow rate of species A entering across boundary 1 is $\dot{m}_1 X_{A,1}$. Using analogous expressions for the other boundaries gives the mass balance for species A,

$$\dot{m}_1 X_{A,1} + \dot{m}_2 X_{A,2} = \dot{m}_3 X_{A,3} + \dot{m}_4 X_{A,4}.$$

Repeating the mass conservation equation for species B gives

$$\dot{m}_1 X_{B,1} + \dot{m}_2 X_{B,2} = \dot{m}_3 X_{B,3} + \dot{m}_4 X_{B,4}.$$

And similarly for species C the mass balance is

$$\dot{m}_1 X_{C,1} + \dot{m}_2 X_{C,2} = \dot{m}_3 X_{C,3} + \dot{m}_4 X_{C,4}.$$

To perform a complete analysis on the system in Figure 2, we would need to know information about the values for flow rates and species mass fractions at the boundary. □