1 Introduction

Power law functions are of the form

$$y = c_1 x^{c_2} \tag{1}$$

where c_1 and c_2 are constants. A power law functions can be transformed to linear relationship by taking the logarithm of Equation (1). The result is still a power law, but in different coordinates. A plot of Equation (1) on log-log axes results in a straight line. Converting Equation (1) to logarithmic coordinates makes some analysis easier.

To see how the logarithmic transformation works, take the log of Equation (1).

$$\begin{aligned}\log(y) &= \log\left(c_1 x^{c_2}\right) \\ &= \log(c_1) + \log\left(x^{c_2}\right) \\ &= \log(c_1) + c_2 \log(x).\end{aligned} \tag{2}$$

Create new variables

$$\tilde{y} = \log(y), \quad \tilde{x} = \log(x),$$

and introduce the new constant

$$k_1 = \log(c_1). \tag{3}$$

Thus, Equation (2) is linear in the log coordinates $(\tilde{x}, \tilde{y})$,

$$\tilde{y} = k_1 + c_2 \tilde{x}$$

To gain familiarity with power law functions, we'll examine some specific cases.

2 Cases where $c_2 > 0$

Figure 1 shows plots of Equation (1) for $c_1 = 1$ and $c_2 = 0.5, 1.0$ and 2.0. The plot on the left side of Figure 1 uses linear axes and the plot on the right side uses log-log axes. In log-log coordinates, Equation (1) is a straight line for any combination of c_1 and c_2.

The trends in Figure 1 hold for any $c_2 > 0$. For $0 < c_2 < 1$, the function increases more slowly as x increases. For $c_2 > 1$, the function increases more rapidly as x increases. You can show how the rate of change depends on c_2 by taking the derivative dy/dx, which is left as an exercise to the reader.

3 Cases where $c_2 < 0$

Figure 2 shows plots of Equation (1) for $c_1 = 15$ and $c_2 = -0.5, -1.0$ and -2.0. The plot on the left side of Figure 2 uses linear axes and the plot on the right side uses log-log axes.

The trends in Figure 2 hold for any $c_2 < 0$. For all $c_2 < 0$, the function decreases more slowly as x increases.

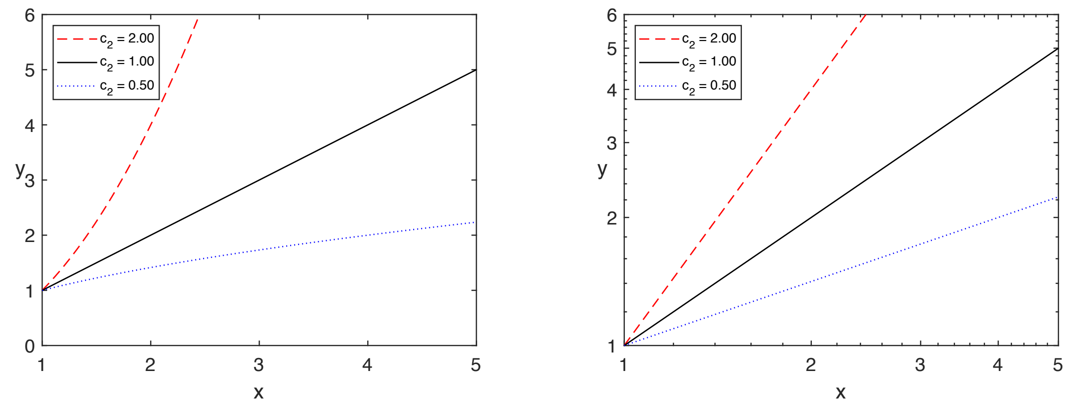

Figure 1: Examples of power law functions, $y = c_1 x^{c_2}$ for cases where $c_2 > 0$. The same functions are plotted on linear axes (left) and log-log axes (right).

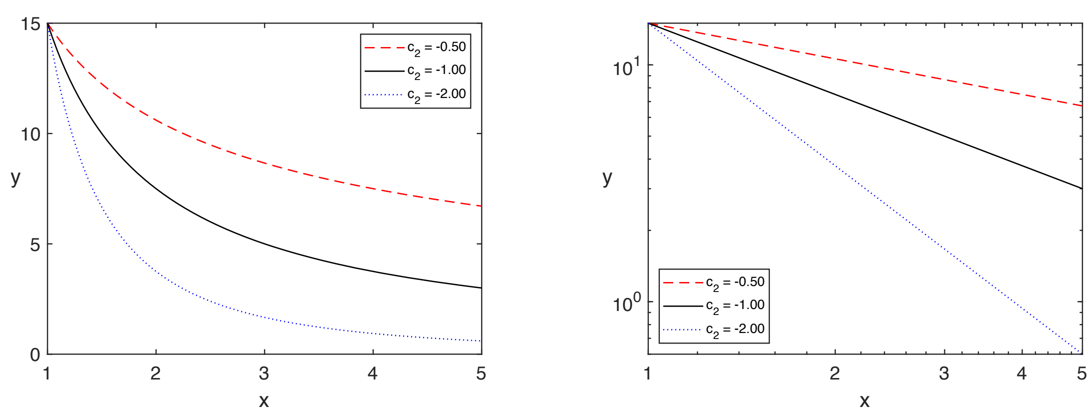

Figure 2: Examples of power law functions, $y = c_1 x^{c_2}$ for cases where $c_2 < 0$. The same functions are plotted on linear axes (left) and log-log axes (right).

4 Shapes depend on combination of c_1, c_2 and range of x

Plots of Equation (1) have different shapes depending on the values of c_1 and c_2, as well as the range of x to be plotted. Figure 3 shows plots of $c_2 > 0$ (left) and $c_2 < 0$ (right) on linear axes.

Unlike the plots in Figure 1 and Figure 2, the functions in Figure 3 are plotted over a range that includes $x < 1$. Since $1^{c_2} = 1$ for any c_2, all of the curves in Figure 3 pass through $(x, y) = (1, 1)$.

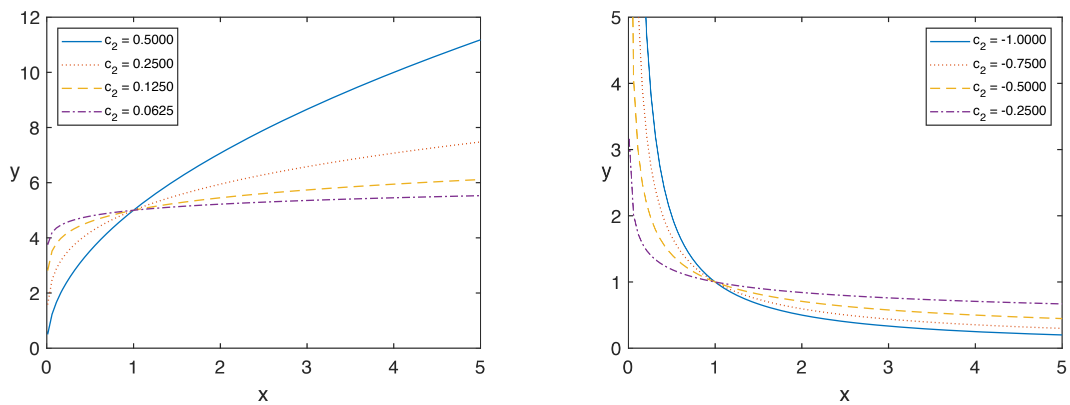

Figure 3: Additional examples of power law functions, $y = c_1 x^{c_2}$. On the left, $c_1 = 5$ and $c_2 > 0$. On the right, $c_1 = 5$ and $c_2 < 0$.